REDUCTION OF LOSSES

IN

AIR-CORED COILS

by

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Of the three basic circuit elements, resistors, capacitors and inductors, inductors are the least perfect. They have relatively the largest residual impedances. This comes about from the very nature of an inductor. It is all the more reason why all possible steps should be taken to reduce these residuals. They are a-c resistance and distributed capacitance, together with its associated dielectric loss. It is the object of this paper to present formulae for the calculation of the losses produced by these residuals and to show how these losses are affected by the physical size of the inductor.

The losses in an air-cored inductor usually have been measured by the a-c resistance of the inductor. At low frequencies the a-c resistance is equal to the d-c resistance. As the frequency is increased a-c resistance increases, due to skin-effect. The current is no longer distributed uniformly over the cross-section of the wire which composes the winding, but is crowded toward the surface of the wire. At high frequencies the current is carried almost entirely in a thin skin near the surface, so the a-c resistance increases greatly.

Formulae have been available for many years for calculating this increase in resistance in straight wires. The fractional increase, until it becomes about 2, varies as the square of the frequency. Beyond this range, it varies as the square root of the frequency and the current is carried in a thin surface layer.

The same method of attack on single layer coils wound with solid wire has been carried out by Butterworth in 1926 and by Palermo and Grover in 1934. Their formulae are quite satisfactory as regards accuracy, but are very awkward to use. Multiple-layer coils and the use of insulated stranded wire (litzendraht) have offered too great complications for any satisfactory solution.

Arguimbau in 1936, in a paper devoted mainly to calculating the losses in iron-cored coils, used a new method of expressing losses in inductors and suggested its application to air-cored coils. Field developed this method in a paper presented at the I.R.E. Convention in 1948 and expanded it at this year's I.R.E. Convention in the Symposium on Basic Circuit Elements.

This new method depends on two old, but neglected, concepts; first, that skin-effect can be considered as the superposition on the uniform current distribution in the winding at low frequencies of the eddy currents induced in the winding by the magnetic field of the coil; second, that the separate coil losses can be expressed in terms of the dissipation factors which they produce and which then can be added to give the total dissipation factor.

The power loss in a round wire of length, l, diameter, d, and resistivity, ρ, produced by a magnetic field of rms flux density, B_c, normal to the wire is

$$P_e = \frac{\pi^3 \, 1 d^4 f^2 B_c^2}{16 \rho 10^{16}} \qquad \text{watt (cm)} \qquad (1)$$

The parallel resistance which represents this power loss when the wire is wound into N turns embracing an area, A, and linked by a magnetic field of rms flux density, B, is

$$R_e = \frac{64 \rho N^2 A^2}{1 d^4} \cdot \frac{B^2}{B_c^2} \qquad \text{ohm (cm)} \qquad (2)$$

The ratio of these flux densities is not calculable and must be determined experimentally.

This resistance is independent of frequency. Hence, the inductor may be represented schematically as in Figure 1 by this resistance, R_e, in parallel with the inductance, L, with its d-c resistance, R_c, in series. The corresponding dissipation factors are

$$D_c = \frac{R_c}{\omega L} = \frac{\alpha}{f} \text{ and } D_e = \frac{\omega L}{R_e} = \beta f \tag{3}$$

The total dissipation factor is their sum, approximately

$$D = D_c + D_e = \frac{\alpha}{f} + \beta f \tag{4}$$

This has a minimum value

$$D_m = 2\sqrt{\alpha \beta} \tag{5}$$

which occurs at a frequency

$$f_m = \sqrt{\frac{\alpha}{\beta}} \tag{6}$$

These two dissipation factors are best represented graphically on a log-log plot as the two 45° lines shown in Figure 2. The copper dissipation factor, D_c, decreases with frequency, while the eddy-current dissipation factor, D_e, increases with frequency. Their sum is a symmetrical curve which has the same shape under all conditions and for all coils. It may, therefore, be drawn by means of a universal template when D_m and f_m are known.

From measurements on some 90 solenoids, both multiple layer and single layer, the ratio of the two flux densities of Equation 2 has been determined and values of D_m and f_m calculated in terms of the coil dimensions shown in Figure 3. For all solenoids

$$D_m = \frac{.81 \, d}{\sqrt{ab}} \tag{7}$$

For multiple layer solenoids

$$f_m = \frac{67}{\delta^2 dcg} \sqrt{\frac{a}{b}} \qquad \text{cycles/sec (in)} \tag{8}$$

while for single layer solenoids

$$f_m = \frac{21.5}{\delta \, d2k} \sqrt{\frac{b}{an}} \qquad \text{cycles/sec(in)} \tag{9}$$

where the spacing factor $\delta = \frac{d}{t}\sqrt{n}$

and where d = diameter of separate strands

n = number of strands

t = outside diameter of stranded wire

The expression for minimum dissipation factor is surprisingly simple, involving merely the strand diameter, d, and a linear dimension which measures the size of the coil. It can be made very small by using #44 wire, whose diameter is about 2 mils, wound on a large coil. When the coil is 4 inches long and has an outside diameter of 4 inches, D_m varies from 0.0008 to 0.0006 as the winding depth, c, is decreased from a maximum value to that holding for a single layer. The frequency for minimum dissipation factor, f_m, increases under these same conditions from 40 kc to 9 Mc for a spacing factor δ of 0.7

All possible loci of the points defined by D_m and f_m are shown in Figure 4 for square coils, $b = d_2$, for values of the ratio $\frac{b}{c}$ from 2, for which the entire coil is filled with wire to single layer, and for outside diameters and lengths from 4 inches to 0.3 inch. Even the smallest coil has a minimum dissipation factor less than 0.01. The exact shape of the coil is not too important. The extremes for coils whose lengths are half and twice their outside diameters are shown by dashed lines. For larger strand diameters the minimum dissipation factors will be increased in proportion to the increase in strand diameter, while the frequency at which it occurs will be correspondingly lowered.

The area covered by the 45° lines of constant values of $\frac{b}{c}$ represents multiple layer coils. For each point there will be a range of inductance values, depending on the number of strands used. For the largest coil, $d_2 = 4$ in and $\frac{b}{c} = 2$, the inductance for 3 strands of #44 wire is 1450 h and for 127 strands 810 mh. For the small coil, $d_2 = 0.3$ in. and $\frac{b}{c} = 2$, the inductance for 3 strands of #44 wire is 3.5 mh and for 127 strands $2.0\,\mu$h.

The area covered by the vertical lines of constant number of strands represents single layer coils. These two areas appear to overlap, but of course do not do so for any particular number of strands. For each point there is now a single value of inductance. For the largest coil and smallest number of strands, $d_2 = 4$ in. and $n = 3$, the inductance is 46 mh, for the smallest coil and largest number of strands, $d_2 = 0.3$ in and $n = 127$, $0.46\,\mu$h. The combined inductance range available in these two areas for multiple-layer and single-layer coils is entirely adequate and the minimum dissipation factors are satisfactorily small. Outside of these areas dissipation factor increases as shown by the curves. Of course, for any one coil the dissipation factor rises symmetrically on either side of its minimum value.

Unfortunately these indicated low values of minimum dissipation factor cannot be realized. Every inductor has associated with it a distributed capacitance, C_0, which gives it a natural frequency

$$f_0 = \frac{159}{\sqrt{LC_0}} \quad \text{(Mc} (\mu h, \mu\mu f) \tag{10}$$

This capacitance has itself a dissipation factor, D_0, resulting from the insulation both of the form on which the coil is wound and of the wire. This produces a third power loss in the coil and contributes a third dissipation factor, D_f, which varies as the square of the frequency.

$$D_f = D_0 \omega^2 L C_0 = D_0\, \frac{f^2}{f_0^2} = \gamma f^2 \tag{11}$$

The total dissipation factor of the coil becomes

$$D = D_c + D_e + D_f = \frac{\alpha}{f} + \beta f + \gamma f^2$$

The complete schematic representation of the inductor is that shown in Figure 5.

Approximate formulae are available for the distributed capacitance of both multiple-layer and single-layer coils, based on the capacitance between layers or between turns. In general single-layer coils have much smaller distributed capacitances that multiple-layer coils. Their values range between 1 and $5\mu\mu$f, being nearly proportional, for square coils, to their diameters. Multiple-layer coils have much larger capacitances, extending sometimes to several hundred micromicrofarads for large coils having only a few layers. All square coils should be bank wound since there are at least twice as many turns per layer as there are layers. For this type of winding the maximum capacitance for $d_2 = 4$ in. is $60\mu\mu$f and decreases for the smaller coils to values comparable with those of single-layer coils.

Measurements on the same 90 solenoids used in developing the eddy-current losses indicate that the dissipation factor, D_O, of this distributed capacitance is about 0.015 for multiple-layer coils and 0.01 for single-layer coils, when wound with insulated stranded wire. These very large values are mainly due to the large dielectric losses of the enamel on the individual strands. What is greatly needed is an enamel with the small dissipation factor of polyethylene.

To observe the effect of this third loss, a square-law line through the point defined by the natural frequency, f_O, and its dissipation factor, D_O, is added to the two 45° lines of Figure 2. The resulting curves for the largest multiple-layer coils, $d_2 = 4$ in. and $n = 127$, are shown at the left of Figure 6 for various values of $\frac{b}{c}$. The minimum dissipation factors are increased by a factor of 3 to 5 and occur at much lower frequencies. The original minimum values are shown as dots. These changes would be greater for smaller numbers of strands because the inductances are larger and the natural frequencies lower. They are less for the smaller coils because there is a smaller ratio of dissipation factor, D_O, of the distributed capacitance and minimum dissipation factor, D_m.

The effect of dielectric losses on dissipation factor is less in single-layer than in multiple-layer coils, because both their inductances and their distributed capacitances are smaller. Two sets of these curves are shown in Figure 6 for 127 and 3 strands for different outside diameters. The lowest minimum dissipation factor is 0.0014 ($Q_m = 700$) for the larger coils, but a value of 0.008 ($Q_m = 120$) can be obtained from a 0.3 in. outside diameter.

In all of these calculations the individual turns have been so spaced as to just not touch adjacent turns, $\delta = 0.7$ for insulated stranded wire. While this is almost a necessity for multiple-layer coils, in single-layer coils the turns may be spaced to almost any extent, certainly to the point where $\delta = 0.1$. This decreases both inductance and distributed capacitance and increases natural frequency. One such curve is shown in Figure 6 for $d_2 = 4$ in and $n = 127$, giving a minimum dissipation factor of 0.0015 at 3 Mc for $22\,\mu h$. If this same inductance were obtained for $d_2 = 0.3$ in and $n = 3$, the minimum dissipation would be 0.008 at 10 Mc. While it appears to be true that large coils will always produce the smallest values of minimum dissipation factor, the smaller coils do not decrease this quantity in proportion to their volume. In addition the small coils will be usable at higher frequencies than the large coils.

So far this entire discussion has been based on the use of stranded insulated wire with the finest individual strands (#44, $d = 0.002$ in). The use of larger strands or large solid wire will increase the minimum dissipation factor except where the dielectric-loss dissipation factor, D_f, has completely overshadowed the eddy-current dissipation factor, D_e. However, large solid wire can be used to advantage in many single-layer coils. The expression for eddy-current dissipation factor, D_e, based on Equation 1, assumes that the magnetic flux which produces the eddy-currents is unaffected by them, that is, that there is no shielding of the interior of the wire by the eddy-currents. As the frequency is raised, such shielding must occur. The magnetic flux no longer passes through the copper wire, but passes around it. The power loss does not increase with the square of the frequency and its dissipation factor, D_e, no longer increases with the frequency. Instead, D_e increases to a maximum and then decreases as the square root of the frequency. This behavior does not often appear in multiple-layer coils, either because the natural frequency of the coil occurs first or because the dielectric loss in the distributed capacitance obscures it. It does appear in single-layer coils wound with solid wire.

The exact shape of the transition curve can not be determined by the methods previously outlined, but the position of the inverse square-root line can. It is best expressed by the coiling factor, λ, of Palermo and Grover. This is the ratio of the a-c resistance of the wire when wound to form the coil to its a-c resistance when straight. The resulting expression for dissipation factor as found from measurements on 80 coils is

$$D_c = \frac{2.0\,(1 + 7\delta d)}{ak\sqrt{\delta f}} \quad \text{(in. cycles)} \tag{13}$$

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for values of δ between 0.1 and 0.7. It is convenient to calculate this quantity at 1 Mc and then to draw an inverse square-law line through that point. It varies inversely with the diameter of the coil and with the square root of the spacing factor δ. The correction factor $7\delta d$ in the numerator is important only for large wires which are closely spaced.

Because this type of eddy-current dissipation factor decreases with increasing frequency, minimum dissipation factor will occur only at approach to the natural frequency. In fact, if the distributed capacitance had no dielectric loss, this minimum would be at the natural frequency. The dissipation factor, D_O, resulting from a steatite form and enameled wire is only 0.004, while the use of polystyrene bar supports and bare wire makes D_O negligible.

The dissipation factor curves for four single-layer solid wire coils are shown at the right in Figure 6 for #24 wire and $D_O = 0.004$. Minimum dissipation factor remains remarkably constant between 0.0025 and 0.0035. Had there been no dielectric loss, minimum dissipation factors at the natural frequencies would have ranged between 0.0012 and 0.0016. The inductances of these coils depend on the diameter, d_2, of the coil and the spacing factor, δ, and range from 1.1 mh for $d_2 = 4$ in. and $\delta = 0.7$ down to 0.01 μh for $d_2 = 0.3$ in. and $\delta = 0.1$. Thus the difference between large and small coils is no longer that of dissipation factor but rather of inductance and operational frequency.

Equivalent Circuit of Inductor
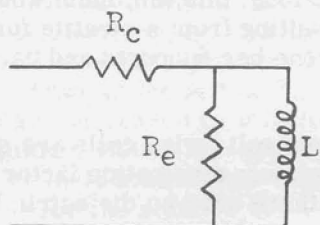

Figure 1

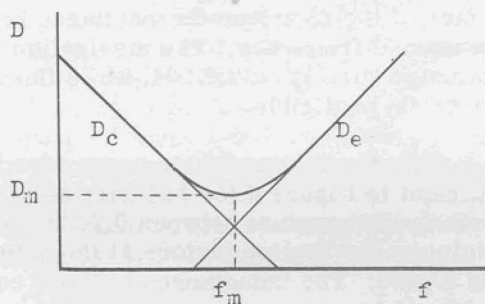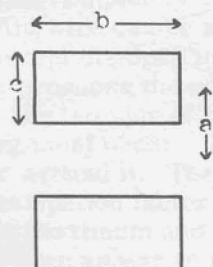## Equal Log-Log Plot of Dissipation Factor

Figure 2

Formulae for Current-sheet Inductance
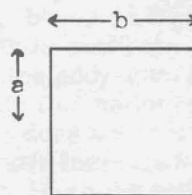
Solenoids

Multiple Layer	Single Layer

Stefan	Nagaoka

$$L \ \mu h(in) \quad .03194 \ a \ N^2 \ g \qquad \frac{.1002 \ a^2 \ N^2 \ k}{b}$$

$$g = f \left(\frac{b}{c}, \frac{c}{a} \right) \qquad k = f \left(\frac{2a}{b} \right)$$
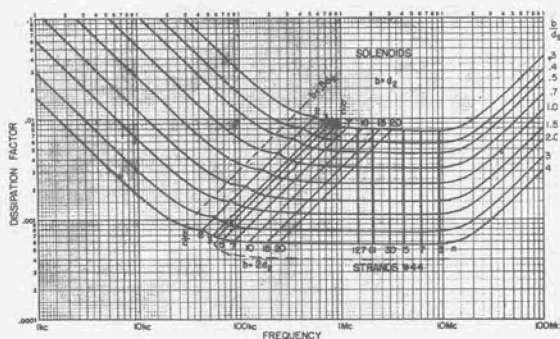
Figure 3

Figure 4

Equivalent Circuit of Inductor

Figure 5

FREQUENCY

Figure 6